

Get going with equations
for simple lists, queues and even ████████

Formal Specifications FOR DUMMIES[®]

Demystifies
everything from
algebraic specifications
to abstraction
functions!

**A Reference
for the
Rest of Us!**

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(that's me)

A simple API

Lists – append, cons and nil:

```
list() ++ list() → list()  
[int() | list()] → list()  
[] → list()
```

How can we test these functions?

Write properties!

Properties in all their wonder

```
prop_appendnil() →  
  ?FORALL(Xs, list(int()),  
  Xs ++ [] == Xs).
```

```
1> eqc:quickcheck(prop:prop_appendnil()).
```

```
.....  
.....
```

```
OK, passed 100 tests
```

But how do we know what properties to write?

QuickSpec to the rescue!

DEMO

listsig.erl

What could you use this for?

Formulating properties

Understanding someone else's code

Finding bugs

Pure functions only!

How does QuickSpec work?

I'm not telling you yet :)

Equations are tested on random input

Equations relate *simple enough* expressions built from functions and variables

Xs	depth 1
Xs ++ []	depth 2
Xs ++ (Ys ++ Zs)	depth 3
Xs ++ ((Ys ++ Zs) ++ Zs)	depth 4

Too
complicate
d

Completeness: all true equations about simple enough expressions are found.

The queue API

Q =

1	2	3
---	---	---

in(4, Q) =

1	2	3	4
---	---	---	---

{head(Q), tail(Q)} = {

1

 ,

2	3
---	---

 }

The queue API, in reverse

Q =

1	2	3
---	---	---

in_r(4, Q) =

4	1	2	3
---	---	---	---

{daeh(Q), liat(Q)} = {

3

 ,

1	2
---	---

 }

Or
liat(Q)!

DEMO

`queuesig.erl`

What's wrong?

`lait(in(X, Q)) /= Q`, because the two queues might have different representations!

But the queues should have the same contents.

In other words,

```
to_list(lait(in(X, Q)) ==  
to_list(Q).
```

Why don't we compare those instead?

What's wrong?

We want to see this law:

```
is_empty(new()) == true()
```

But QuickSpec doesn't know about true!

What's wrong?

We want to see this law:

$$\text{reverse}(\text{to_list}(Q)) == \text{to_list}(\text{reverse}(Q))$$

In that case, we need the `reverse` function on lists!

How did we get nice laws?

Compare the elements of a queue, not its internal representation:

```
observe(Q, queue) ->  
  queue:to_list(Q).
```

Add `reverse` so that we see the symmetry between `tail` and `lait`.

Use lists as a model of queues.

What on earth is this?

`bar(X, foo()) == apa()`

`bar(X, monkey(X, P, I)) == P`

`bar(Y, monkey(X, P, foo())) == bar(X, monkey(Y, P, foo()))`

`bar(X, monkey(Y, apa(), foo())) == apa()`

`monkey(X, P, monkey(X, Q, I)) == monkey(X, P, I)`

`monkey(Y, P, monkey(X, P, I)) == monkey(X, P, monkey(Y, P, I))`

...and why does the highlighted law hold?

...it's arrays!

```
get(I,new()) == default_element()
```

```
get(I,set(I,X,A)) == X
```

```
get(J,set(I,X,new())) == get(I,set(J,X,new()))
```

```
get(I,set(J,default_element(),new())) == new()
```

```
set(I,X,set(I,Y,A)) == set(I,X,A)
```

```
set(J,X,set(I,X,A)) == set(I,X,set(J,X,A))
```

What on earth is this?

`bar(X, foo()) == apa()`

`bar(X, monkey(X, P, I)) == P`

`bar(Y, monkey(X, P, foo())) == bar(X, monkey(Y, P, foo()))`

`bar(X, monkey(Y, apa(), foo())) == apa()`

`monkey(X, P, monkey(X, Q, I)) == monkey(X, P, I)`

`monkey(Y, P, monkey(X, P, I)) == monkey(X, P, monkey(Y, P, I))`

...and why does the highlighted law hold?

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Regular expressions

Things that match strings.

abc matches the string “abc”

ab*c matches “abbbbbbbbc”

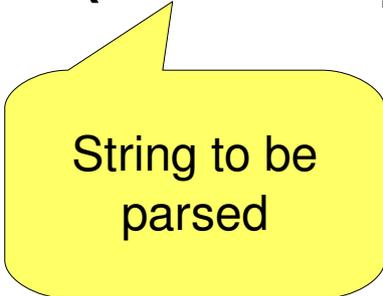
a(b|c)* matches “abccccbbbc”

ab+ matches “abb” but not “a”

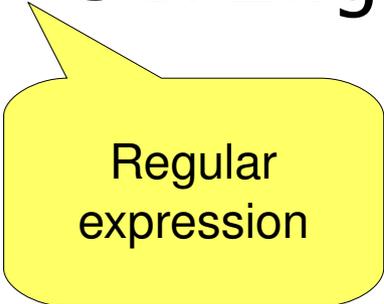
a? matches only “a” or “”

Regular expression functions

`run(string, string) → boolean`



String to be
parsed



Regular
expression

Too unstructured!

Regular expression operators

`run(string, regex) → boolean`

`star(regex) → regex`

`char(char) → regex`

`any_char star(R) ->`

`concat(char, star(R)) → regex`

`choice(regex, regex) → regex`

For example,

`concat(char($a), star(char($b)))`

A regex is
still a
string
really

star(R) ->
“(“ ++ R ++ “)*”.

Are two regular expressions equal?

Easiest way to find out: test on random input

First try:

```
observe(R, regex) →  
  re:run(..., R).
```

Are two regular expressions equal?

Easiest way to find out: test on random input

Second try:

```
observe(R, regex, S) →  
  re:run(S, R).
```

```
context() →  
  list(char()).
```

OK, let's try it out!

```
2> laws:laws(re_sig).  
Classifying terms of depth 0... 2 terms....  
2 classes.  
Classifying terms of depth 1... 10 terms....  
10 classes.  
Classifying terms of depth 2... 78 terms.....  
<<computer goes into a sulk>>
```

What could be wrong?

DEMO

re_sig.erl

The killer regular expression

```
2> re:run("abc", "((a|())+|a)+").  
<<computer goes into a sulK>>
```

For now: remove * and +.

Are two regular expressions equal?

Input data is too random!

```
re:run("cpj sd~!!££$", R).
```



This will
probably
return
nomatch

Test on strings of as and bs.

The story so far

Well, it all seems to work OK...

But what about *?

I don't want to fix PCRE

In principle we could avoid dodgy regular expressions

Let's switch to a non-breaky regular expression library instead :)

DEMO

nfa_re_sig.erl

It's a bug!

$$R^*;S^* = (R|S)^*$$

RRRRRSSS

RSRRSSR

$$R|S^* = (R|S)^*$$

R
or
SSSSS

RSRRSSR

What could we use this for

Understanding a library (fixed-point arithmetic)

Finding bugs

Extracting tests

Optimisation?

The paper

Lots of examples (Erlang and Haskell)

Case studies: binary heaps and fixed-point arithmetic

How everything works

How does QuickSpec work?

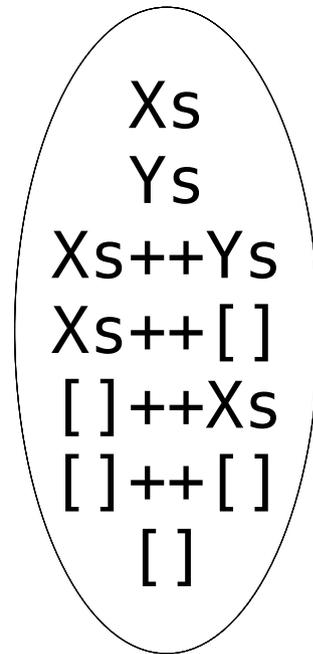
Generate all expressions up to a given depth:

Xs
Ys
Xs++Ys
Xs++[]
[]++Xs
[]++[]
[]

We assume that two expressions are equal until we find a counterexample.

How does QuickSpec work?

Pick some random values for testing:

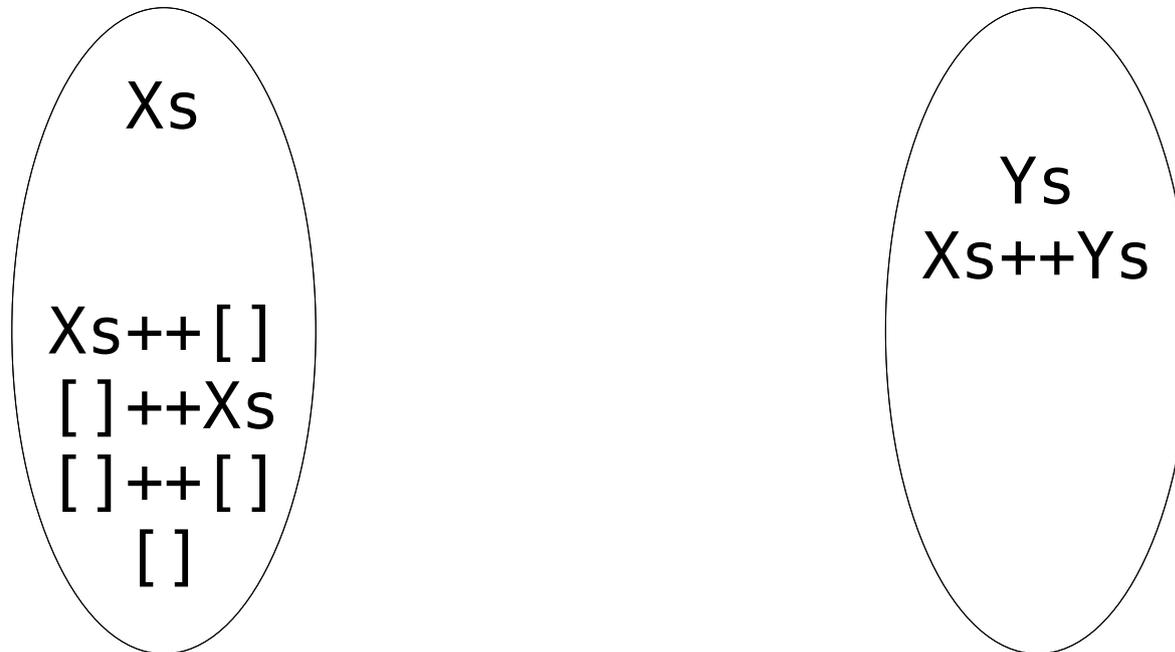


Xs
Ys
Xs++Ys
Xs++[]
[]++Xs
[]++[]
[]

Xs = [], Ys = [1]

How does QuickSpec work?

Pick some random values for testing:



$Xs = [1], Ys = []$

How does QuickSpec work?

Produce some equations:

$[]++[]$
 $[]$

Xs
 $Xs++[]$
 $[]++Xs$

Ys

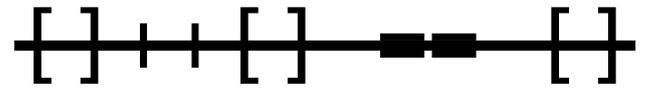
$Xs++Ys$

$[]++[] == []$

$Xs++[] == Xs$
 $[]++Xs == Xs$

How does QuickSpec work?

Filter out the redundant equations (then print the rest):



The diagram shows a sequence of four equations: $xs++xs == xs$, $xs++xs == xs$, $xs++xs == xs$, and $xs++xs == xs$. A horizontal line is drawn through the first two equations, and two small black squares are placed on the line above the third equation, indicating that the first two equations are filtered out as redundant.

$$\begin{array}{l} \cancel{xs++xs == xs} \\ \cancel{xs++xs == xs} \\ xs++xs == xs \\ xs++xs == xs \end{array}$$

This is the hard bit!

Caveats

Your functions need to be pure and terminating

You might need to include some auxiliary functions

observe is quite delicate

```
observe(Q, queue) →  
  queue: len(Q).
```

```
in_r(X, Q) == in_r(Y, Q)
```

```
head(in_r(X, Q)) /= head(in_r(Y, Q))
```

Should be able to spot this automatically.

What's to come next?

Testing imperative modules

Conditional equations:

$I \neq J \implies$

$\text{set}(I, X, \text{set}(J, Y, A)) ==$
 $\text{set}(J, Y, \text{set}(I, X, A))$

(demo: Haskell arrays)